AP Calculus BC

Partial Fraction Decomposition

1) $\frac{15x+93}{x^2+13x+40} = \frac{A}{x+5} + \frac{B}{x+8}$

$A(x+8) + B(x+5) = 15x+93$

$x=-8$	$x=-5$
$-3B=-27$	$3A=18$
$B=9$	$A=6$

$\frac{6}{x+5} + \frac{9}{x+8}$

2) $\frac{4}{x^2-8x-20} = \frac{A}{x-10} + \frac{B}{x+2}$

$A(x+2) + B(x-10) = 4$

$x=-2$	$x=10$
$-12B=4$	$12A=4$
$B=-\frac{1}{3}$	$A=\frac{1}{3}$

$\frac{\frac{1}{3}}{x-10} + \frac{-\frac{1}{3}}{x+2}$

3) $\frac{15x+75}{(x+8)(x+3)} = \frac{A}{x+8} + \frac{B}{x+3}$

$A(x+3) + B(x+8) = 15x+75$

$x=-3$	$x=-8$
$5B=30$	$-5A=-45$
$B=6$	$A=9$

$\frac{9}{x+8} + \frac{6}{x+3}$

4) $\frac{9x-17}{(x^2-4x+3)} = \frac{A}{x-3} + \frac{B}{x-1}$

$A(x-1) + B(x-3) = 9x-17$

$x=1$	$x=3$
$-2B=-8$	$2A=10$
$B=4$	$A=5$

$\frac{5}{x-3} + \frac{4}{x-1}$

5) $\int \frac{1}{64-121x^2}\,dx$

$\frac{1}{64-121x^2} = \frac{A}{8-11x} + \frac{B}{8+11x}$

$A(8+11x) + B(8-11x) = 1$

$x=-\frac{8}{11}$	$x=\frac{8}{11}$
$16B=1$	$16A=1$
$B=\frac{1}{16}$	$A=\frac{1}{16}$

$\int \left[\frac{\frac{1}{16}}{8-11x} + \frac{\frac{1}{16}}{8+11x}\right] dx$

$\frac{\frac{1}{16}\ln(8-11x)}{-11} + \frac{\frac{1}{16}\ln(8+11x)+C}{11}$

6) $\int \frac{9}{x(x+3)}\,dx = \int \left[\frac{A}{x} + \frac{B}{x+3}\right] dx$

$A(x+3) + Bx = 9$

$x=-3$	$x=0$
$-3B=9$	$3A=9$
$B=-3$	$A=3$

$\int \left[\frac{3}{x} - \frac{3}{x+3}\right] dx$

$\boxed{3\ln|x| - 3\ln|x+3| + C}$

7) $\int \frac{11\,dx}{2x^2-9x-5} = \int \left[\frac{A}{(2x+1)} + \frac{B}{(x-5)}\right] dx$

$A(x-5) + B(2x+1) = 11$

$x=5$	$x=-\frac{1}{2}$
$11B = 11$	$-\frac{11}{2}A = 11$
$B = 1$	$A = -2$

$$\left[\frac{-2}{2x+1} + \frac{1}{x-5}\right] dx$$

$$\boxed{-\ln|2x+1| + \ln|x-5| + C}$$

8) $\int_{1/2}^{1} \frac{2y+3}{y^2+y}\,dy = \int_{1/2}^{1} \frac{2y+3}{(y+1)y}$

$$\frac{A}{y+1} + \frac{B}{y} = \frac{2y+3}{(y+1)y}$$

$$Ay + B(y+1) = 2y+3$$

$y=0$	$y=-1$
$B = 3$	$-A = 3$
	$A = -3$

$$\int_{1/2}^{1} \left[-\frac{3}{y+1} + \frac{3}{y}\right] dy$$

$$\left(-3\ln(y+1) + 3\ln y\right)\Big|_{1/2}^{1}$$

$$(-3\ln 2 + 0) - (-3\ln 3/2 + 3\ln 1/2)$$

$$-3\ln 2 + 3\ln 3 + 3\ln 2 - 3\ln 1 - 3\ln 2$$

$$3\ln 3 - 3\ln 2$$

$$\boxed{3\ln 3/2}$$

9) $\int_4^5 \frac{4}{(x-1)(x+3)}\,dx$

$$\frac{A}{x-1} + \frac{B}{x+3} = \frac{4}{(x-1)(x+3)}$$

$A(x+3) + B(x-1) = 4$

$x=-3$	$x=1$
$-4B = 4$	$4A = 4$
$B = -1$	$A = 1$

$$\int_4^5 \left[\frac{1}{x-1} - \frac{1}{x+3}\right] dx$$

$$\ln(x-1) - \ln(x+3)\Big|_4^5$$

$$(\ln 4 - \ln 8) - (\ln 3 - \ln 7)$$

$$\ln 1/2 - \ln 3/7$$

$$\boxed{\ln \frac{14}{3}}$$